

Section B – Surds and Graphs

Surds

Exercise 1

Work out the following. Leave the answer in surd form where appropriate.

1). $\sqrt{2} \times \sqrt{3}$ 2). $\sqrt{5} \times \sqrt{2}$ 3). $\sqrt{7} \times \sqrt{3}$ 4). $\sqrt{8} \times \sqrt{2}$ 5). $\sqrt{3} \times \sqrt{3}$
6). $\sqrt{7} \times \sqrt{5}$ 7). $\sqrt{3} \times \sqrt{5}$ 8). $\sqrt{5} \times \sqrt{6}$ 9). $\sqrt{3} \times \sqrt{11}$ 10). $\sqrt{2} \times \sqrt{18}$
11). $\sqrt{2} \times \sqrt{7}$ 12). $\sqrt{5} \times \sqrt{5}$ 13). $\sqrt{5} \times \sqrt{11}$ 14). $\sqrt{3} \times \sqrt{13}$ 15). $\sqrt{7} \times \sqrt{6}$

Exercise 2

Expand the brackets:

1) $\sqrt{2}(3 - \sqrt{2})$ 2) $\sqrt{2}(3 - 2\sqrt{2})$ 3) $\sqrt{3}(\sqrt{27} - 1)$ 4) $(\sqrt{2} - 1)(\sqrt{2} + 1)$
5) $(\sqrt{3} - 2)(\sqrt{3} - 1)$ 6) $(2\sqrt{2} + 1)(\sqrt{2} - 2)$ 7) $(3\sqrt{3} - 2)(3\sqrt{3} + 2)$ 8) $(2\sqrt{5} + 3)(3\sqrt{5} - 2)$
9) $(\sqrt{3} - 1)(\sqrt{2} + 1)$ 10) $(2\sqrt{6} - 3)^2$ 11) $(\sqrt{x} - 1)(\sqrt{x} + 1)$ 12) $(2\sqrt{x} - 1)^2$

Removing a Square Factor

Express these surds in the form $a\sqrt{b}$.

1). $\sqrt{8}$ 2). $\sqrt{27}$ 3). $\sqrt{20}$ 4). $\sqrt{32}$ 5). $\sqrt{80}$
6). $\sqrt{44}$ 7). $\sqrt{75}$ 8). $\sqrt{72}$ 9). $\sqrt{45}$ 10). $\sqrt{108}$
11). $\sqrt{28}$ 12). $\sqrt{125}$ 13). $\sqrt{245}$ 14). $\sqrt{192}$ 15). $\sqrt{405}$
16). $\sqrt{112}$ 17). $\sqrt{63}$ 18). $\sqrt{180}$ 19). $\sqrt{99}$ 20). $\sqrt{48}$

Rationalising the Denominator

1). $\frac{3}{\sqrt{3}}$ 2). $\frac{10}{\sqrt{5}}$ 3). $\frac{21}{\sqrt{7}}$ 4). $\frac{8}{\sqrt{2}}$ 5). $\frac{24}{\sqrt{6}}$
6). $\frac{1}{\sqrt{3}}$ 7). $\frac{1}{\sqrt{2}}$ 8). $\frac{1}{\sqrt{5}}$ 9). $\frac{2}{\sqrt{3}}$ 10). $\frac{9}{\sqrt{15}}$
11). $\frac{21}{\sqrt{6}}$ 12). $\frac{8}{\sqrt{18}}$ 13). $\frac{2}{\sqrt{5}}$ 14). $\frac{9}{\sqrt{6}}$ 15). $\frac{30}{\sqrt{75}}$
16). $\frac{\sqrt{12}}{\sqrt{50}}$ 17). $\frac{\sqrt{12}}{\sqrt{3}}$ 18). $\frac{3\sqrt{2}}{\sqrt{10}}$ 19). $\frac{3\sqrt{7}}{\sqrt{21}}$ 20). $\frac{4\sqrt{5}}{\sqrt{20}}$

Sketching Graphs

Sketch the following graphs:

(1) $y = x^2$ (2) $y = x^3$ (3) $y = \frac{1}{x}$ (4) $y = 3^x$ (5) $y = \sin x$ (6) $y = \cos x$ (7) $x^2 + y^2 = 25$

Transformations of Graphs

1. Sketch the graphs of the following curves using appropriate transformations of $y = x^2$.

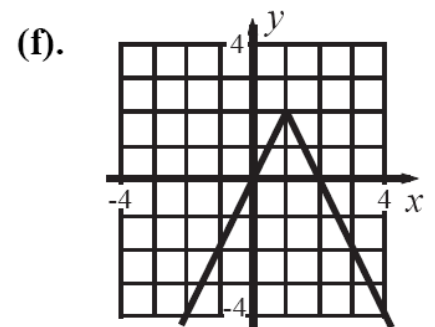
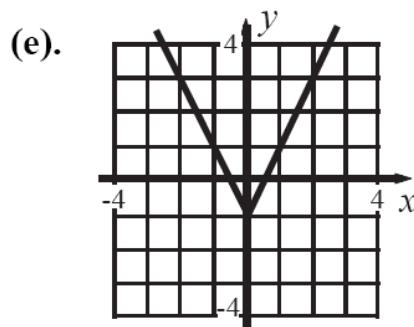
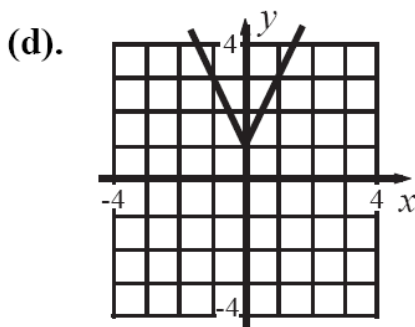
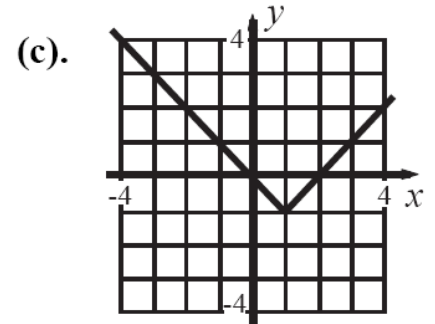
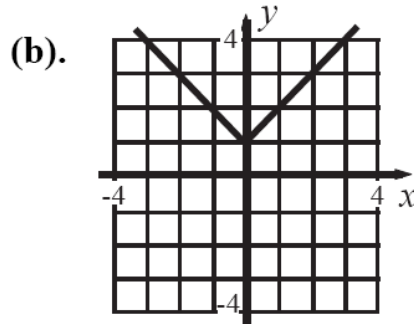
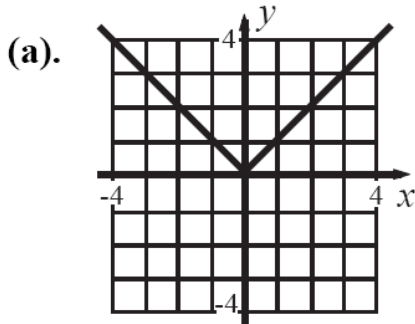
(a) $y = (x-1)^2 + 3$ (b) $y = 2x^2 - 1$ (c) $y = \frac{1}{2}x^2 + 2$

(d) $y = 2(x+3)^2$ (e) $y = 1 - x^2$ (f) $y = -(x-1)^2$



2. The diagram in (a), below, shows the graph of a function $y = f(x)$.

Write down the equations of the functions in parts (b) to (f) in terms of $f(x)$.



5. Sketch the graphs of the following curves using appropriate transformations of $y = \sin x$.

(a) $y = \sin x + 1$ (b) $y = \sin 2x - 1$ (c) $y = 2 \sin x + 2$

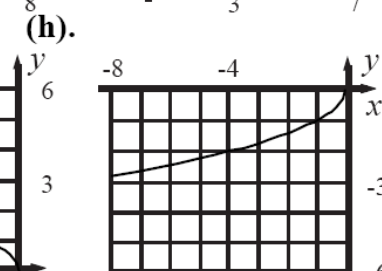
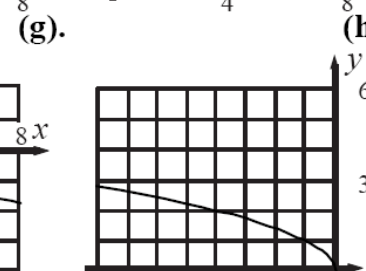
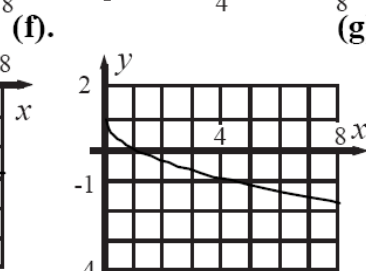
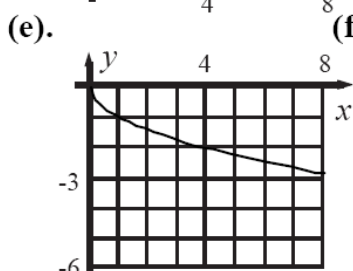
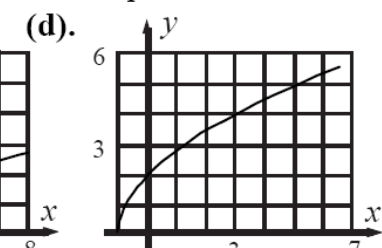
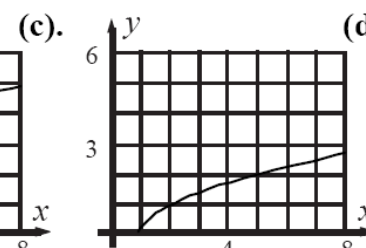
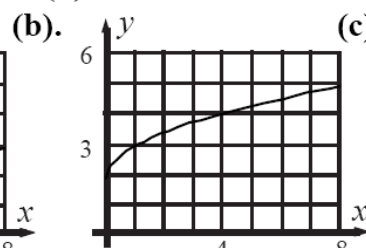
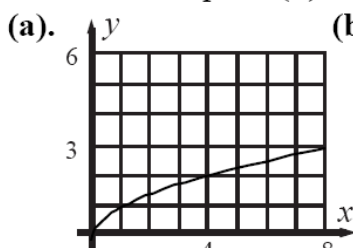
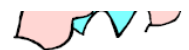
(d) $y = 1 - \sin x$ (e) $y = \sin(x - 90^\circ)$ (f) $y = 2 \sin(x + 90^\circ) - 2$

(g) $y = 2 \sin 3x - 1$ (h) $y = \sin(3x - 90^\circ)$ (i) $y = 1 - \sin(-x)$



6. The graph in (a) below shows the function $y = \sqrt{x}$.

The curves in parts (b) to (h) are transformations of this function. Find their equations.



$y = mx + c$

Exercise 1

For each of the following, state the gradient and intercept and find the coordinates of the point where they cross the x axis:

- | | | | | |
|------------------|------------------|--------------------|--------------------|-----------------------|
| 1) $y = 4x + 2$ | 2) $y = 3x - 5$ | 3) $y = -2x + 7$ | 4) $y = -6x - 2$ | 5) $y = x - 1$ |
| 6) $y = 3 + 5x$ | 7) $y = 7 - 3x$ | 8) $y = -5 + 4x$ | 9) $y = -2 - 9x$ | 10) $y = 5 - x$ |
| 11) $y + 5 = 3x$ | 12) $y - 4x = 2$ | 13) $2y = 6x + 10$ | 14) $3y - 12x = 9$ | 15) $2x - 5y + 8 = 0$ |

Exercise 2

- 1) Write down the equation of the line which is parallel to $y = 5x - 3$ and passes through (0,7)
- 2) A line has gradient 2. Find the equation of the line which is perpendicular to it and passes through (0,5).
- 3) The equations of two lines are $y = 2x - 3$ and $3x + 6y = 9$. Are they perpendicular?
- 4) A line has gradient 3 and passes through (1,4) Find the equation of the line.
- 5) A line passes through (4,5) and is perpendicular to $y = 2x - 5$. Find the equation of the line.

ANSWERS

Section B – Surds and Graphs

Surds

Exercise 1

- | | | | | |
|------------------|-----------------|------------------|------------------|------------------|
| 1). $\sqrt{6}$ | 2). $\sqrt{10}$ | 3). $\sqrt{21}$ | 4). 4 | 5). 3 |
| 6). $\sqrt{35}$ | 7). $\sqrt{15}$ | 8). $\sqrt{30}$ | 9). $\sqrt{33}$ | 10). 6 |
| 11). $\sqrt{14}$ | 12). 5 | 13). $\sqrt{55}$ | 14). $\sqrt{39}$ | 15). $\sqrt{42}$ |

Exercise 2

- | | | | | | | |
|---------------------|---|-----------------------|-------------|--------------------------|--------------------|-------|
| 1) $3\sqrt{2} - 2$ | 2) $3\sqrt{2} - 4$ | 3) $9 - \sqrt{3}$ | 4) 1 | 5) $5 - 3\sqrt{3}$ | 6) $2 - 3\sqrt{2}$ | 7) 23 |
| 8) $24 + 5\sqrt{5}$ | 9) $\sqrt{6} + \sqrt{3} - \sqrt{2} - 1$ | 10) $33 - 12\sqrt{6}$ | 11) $x - 1$ | 12) $4x + 1 - 4\sqrt{x}$ | | |

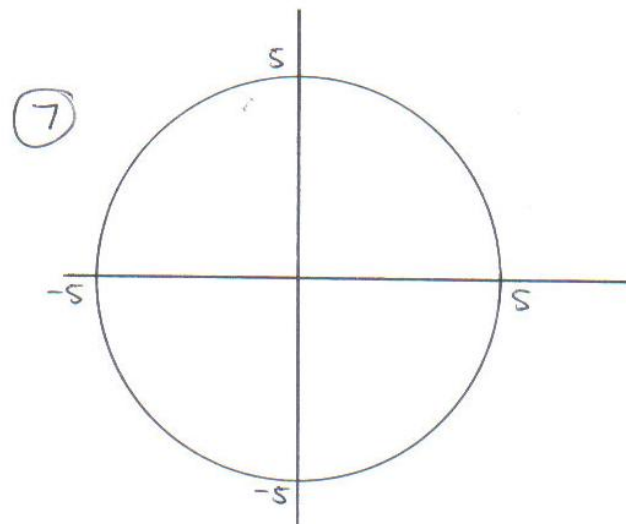
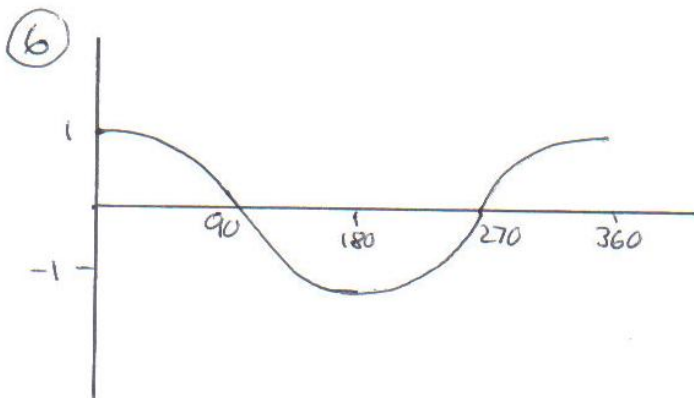
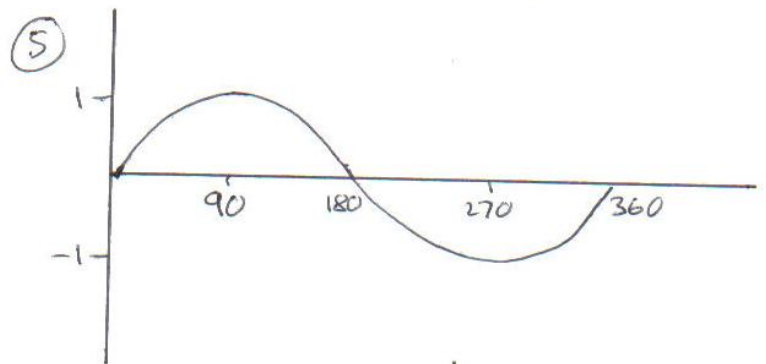
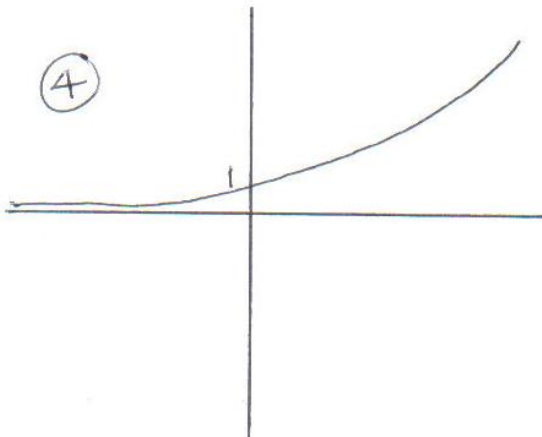
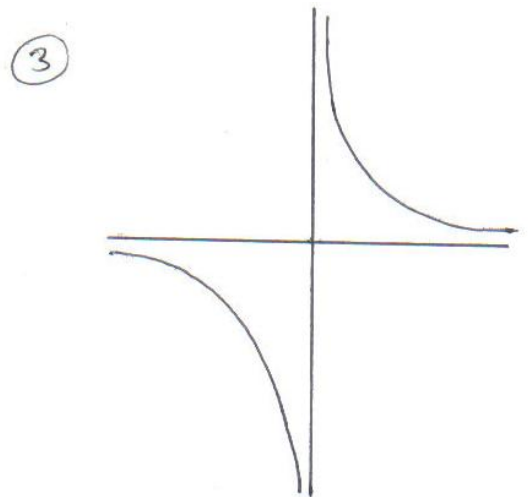
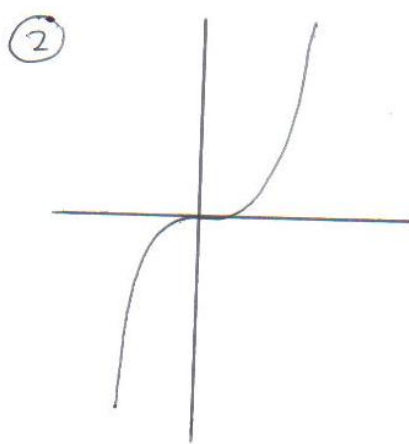
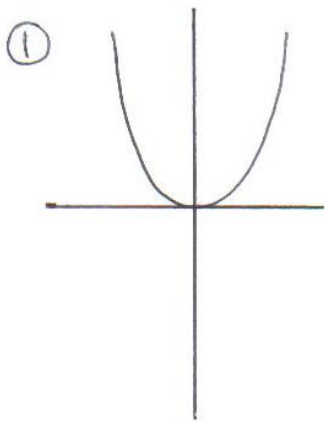
Removing a Square Factor

- | | | | | |
|------------------|------------------|------------------|-------------------|------------------|
| 1). $2\sqrt{2}$ | 2). $3\sqrt{3}$ | 3). $2\sqrt{5}$ | 4). $4\sqrt{2}$ | 5). $4\sqrt{5}$ |
| 6). $2\sqrt{11}$ | 7). $5\sqrt{3}$ | 8). $6\sqrt{2}$ | 9). $3\sqrt{5}$ | 10). $6\sqrt{3}$ |
| 11). $2\sqrt{7}$ | 12). $5\sqrt{5}$ | 13). $7\sqrt{5}$ | 14). $8\sqrt{3}$ | 15). $9\sqrt{5}$ |
| 16). $4\sqrt{7}$ | 17). $3\sqrt{7}$ | 18). $6\sqrt{5}$ | 19). $3\sqrt{11}$ | 20). $4\sqrt{3}$ |

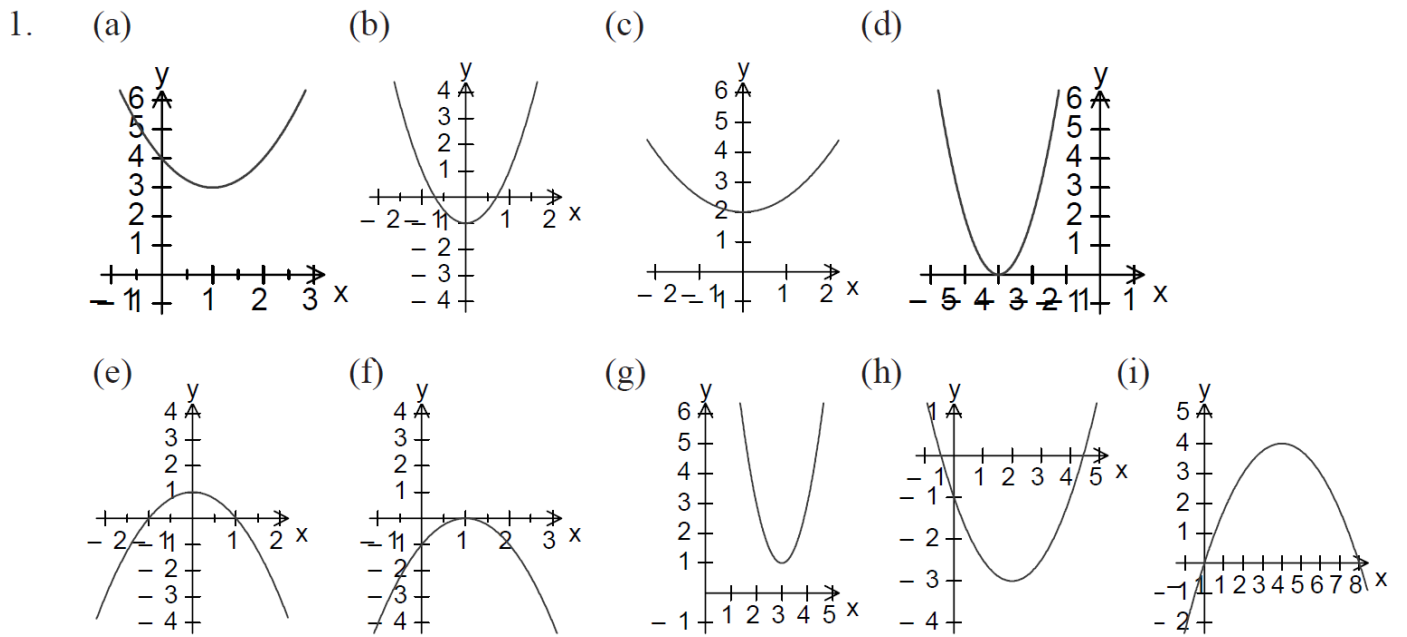
Rationalising the Denominator

- | | | | | |
|--------------------|--------------------|--------------------|--------------------|---------------------|
| 1). $\sqrt{3}$ | 2). $2\sqrt{5}$ | 3). $3\sqrt{7}$ | 4). $4\sqrt{2}$ | 5). $4\sqrt{6}$ |
| 6). $\sqrt{3/3}$ | 7). $\sqrt{2/2}$ | 8). $\sqrt{5/5}$ | 9). $2\sqrt{3/3}$ | 10). $3\sqrt{15/5}$ |
| 11). $7\sqrt{6/2}$ | 12). $4\sqrt{2/3}$ | 13). $2\sqrt{5/5}$ | 14). $3\sqrt{6/2}$ | 15). $2\sqrt{3}$ |
| 16). $\sqrt{6/5}$ | 17). 2 | 18). $3\sqrt{5/5}$ | 19). $\sqrt{3}$ | 20). 2 |

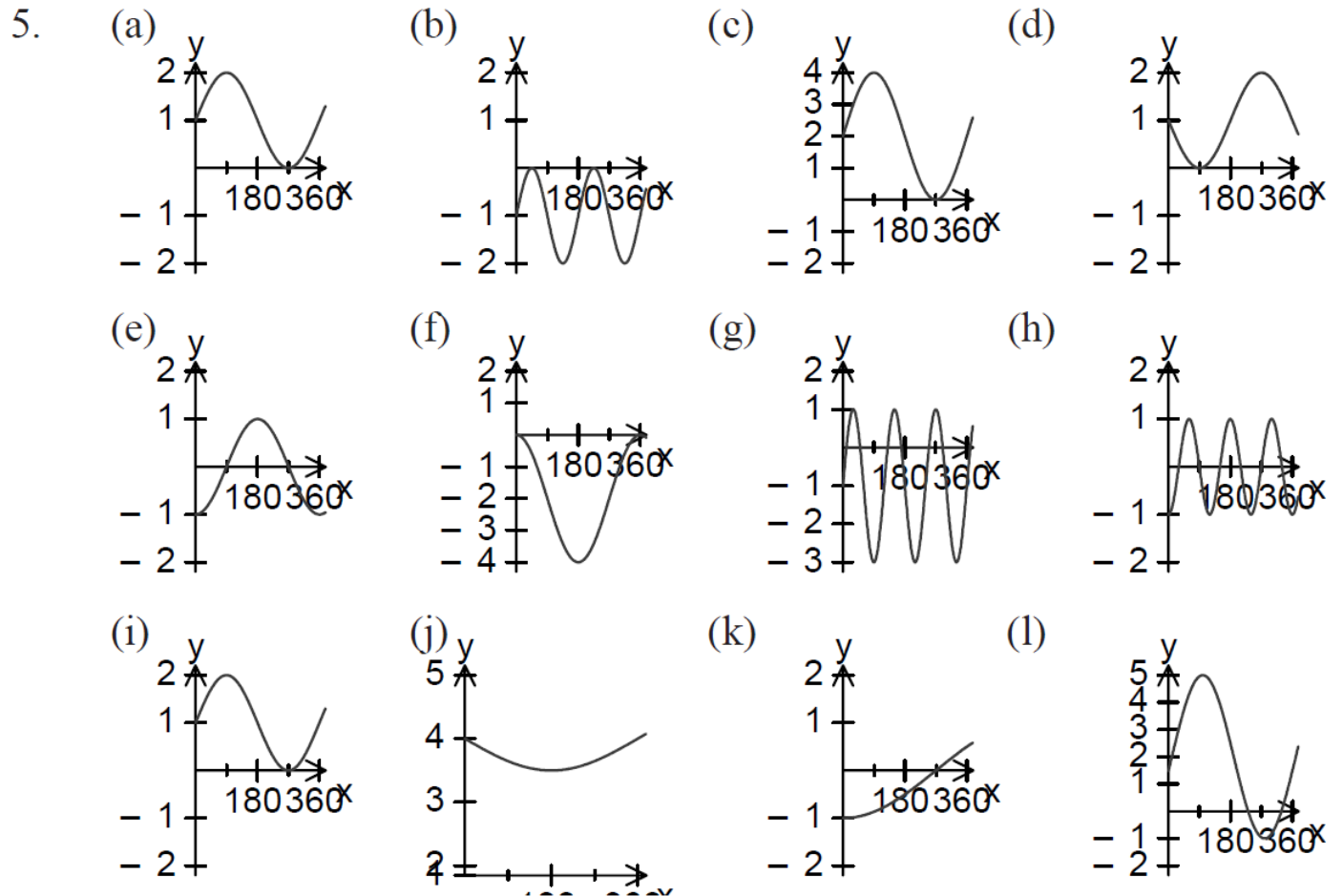
Sketching Graphs



Transformations of Graphs



2. (b) $y = f(x) + 1$ (c) $y = f(x-1) - 1$ (d) $y = 2f(x) + 1$ (e) $y = 2f(x) - 1$ (f) $y = 2 - 2f(x - 1)$



6. (b) $y = \sqrt{x} + 2$ (c) $y = \sqrt{x-1}$ (d) $y = 2\sqrt{x+1}$ (e) $y = -\sqrt{x}$
 (f) $y = 1 - \sqrt{x}$ (g) $y = \sqrt{-x}$ (h) $y = -\sqrt{-x}$

$$y = mx + c$$

Exercise 1

- | | | | | |
|--------------------|--------------------|--------------------|---------------------|----------------------|
| 1) 4, 2, (-0.5,0) | 2) 3, -5, (5/3,0) | 3) -2, 7, (3.5,0) | 4) -6, -2, (-1/3,0) | 5) 1, -1, (1,0) |
| 6) 5, 3, (-3/5,0) | 7) -3, 7, (7/3,0) | 8) 4, -5, (5/4,0) | 9) -9, -2, (-2/9,0) | 10) -1, 5, (5,0) |
| 11) 3, -5, (5/3,0) | 12) 4, 2, (-1/2,0) | 13) 3, 5, (-5/3,0) | 14) 4, 3, (-3/4,0) | 15) 2/5, 8/5, (-4,0) |

Exercise 2

- | | | | | |
|-----------------|----------------------------|--------|-----------------|----------------------------|
| 1) $y = 5x + 7$ | 2) $y = -\frac{1}{2}x + 5$ | 3) Yes | 4) $y = 3x + 1$ | 5) $y = -\frac{1}{2}x + 7$ |
|-----------------|----------------------------|--------|-----------------|----------------------------|